

Annual Report, 2000 Strain Rate Modeling

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We made further development of a method to optimally estimate strains or strain rates from discrete geodetic data and applied the method to model the Southern California Crustal Motion Map data. Unlike some of the previous methods, our method does not require the deformation field to be “homogeneous” nor “isotropic”. According to our method, geodetic displacement data are reweighted and fit to a model of continuous deformation from which strains are derived. The reweighting is done in such a way that data measured closer to a spot under evaluation contribute more to the estimation. The reweighting determines the degree of smoothing and the uncertainties of the strain estimates. An optimal solution can be obtained by balancing trade-offs between the standard deviations of strain estimates and the weighting attributed to the data.

Method

Shen et al. (1996) [10] introduced a method to model strains as continuous functions using a modified least-squares method. At each location \underline{R} , assuming a uniform strain field, the strains and the geodetic data can be linked by a linear relationship:

$$\underline{d} = A \underline{m} + \underline{e} \quad (1)$$

where \underline{d} is the data vector, \underline{m} the vector for the unknowns of translation, rotation, and strains, A the partial derivative matrix, and \underline{e} the error vector. For a case of horizontal strains, $\underline{m} = (U_x \ U_y \ t_{xx} \ t_{xy} \ t_{yy} \ ?)^t$ in a Cartesian coordinate. U_x and U_y are the translation components in x and y directions respectively, t_{xx} t_{xy} and t_{yy} are the horizontal strain components respectively, and $?$ is the rotation.

If the input geodetic data are station velocities, equation (1) can be written as:

$$\begin{bmatrix} V_{x_1} \\ V_{y_1} \\ V_{x_2} \\ V_{y_2} \\ \dots \\ V_{x_n} \\ V_{y_n} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta x_1 & \Delta y_1 & 0 & \Delta y_1 \\ 0 & 1 & 0 & \Delta x_1 & \Delta y_1 & -\Delta x_1 \\ 1 & 0 & \Delta x_2 & \Delta y_2 & 0 & \Delta y_2 \\ 0 & 1 & 0 & \Delta x_2 & \Delta y_2 & -\Delta x_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & \Delta x_n & \Delta y_n & 0 & \Delta y_n \\ 0 & 1 & 0 & \Delta x_n & \Delta y_n & -\Delta x_n \end{bmatrix} \begin{bmatrix} U_x \\ U_y \\ t_{xx} \\ t_{xy} \\ t_{yy} \\ w \end{bmatrix} + \begin{bmatrix} e_{x1} \\ e_{y1} \\ e_{x2} \\ e_{y2} \\ \dots \\ e_{xn} \\ e_{yn} \end{bmatrix} \quad (2)$$

where V_{x_i} and V_{y_i} are the i -th observed station velocity components at location \underline{r}_i . All the variables on the right hand side of the equation are to be evaluated at a location \underline{R} , $?_{x_i}$ and $?_{y_i}$ are the vector components of $? \underline{R}_i = \underline{r}_i - \underline{R}$. Let C be the covariance matrix of the velocity data, we reconstruct the covariance matrix for error \underline{e} by multiplying a weighting factor to each of its components. The weighting is given as

$$C_{pq} = \sum_{i,j} B_{ij} \quad (3)$$

Where $B_{ij} = \exp(-R_{ij}^2/D^2)$ and $R_{ij}^2 = R_i^2 + DR_j^2$, i and j are the indices of stations whose velocity components correspond to the p -th and q -th components of matrix C_{pq} respectively. D is a smoothing distance parameter, which determines the amount of smoothing around a given spot. Equation (2) is solved using least squares with the revised covariance matrix.

In Shen et al. (1996) the smoothing distance D is considered as a fixed constant for a given region. This is adequate if the network distribution is approximately uniform and the data uncertainties are in the same range. However, such conditions are not satisfied sometimes. More often than not stations are densely populated in one area and sparsely distributed in another. A fixed D may provide too much smoothing for the former and less for the latter. An optimal employment of the smoothing distance D according to the *in situ* data strength is needed.

Here we introduce a method to optimally estimate the smoothing distance D and provide a means to quantitatively assess the goodness of the interpolation. At each spot under evaluation, we compute a suite of solutions as a function of the smoothing distance D . For each solution we sum up the diagonal terms of the weighting matrix B and call it gross weighting W . When D approaches infinity, W goes to zero, and the uncertainty σ for the strain estimate also go to infinity because the signal to noise ratio for the input data are close to zero. It is vice versa when D approaches zero. An optimal solution can be obtained by locating a point in this trade-off curve between W and σ with the smallest “distance” to the origin.

We applied our method to the v2.0 release of the Southern California Crustal Motion Map (http://www.scecdc.scec.org/group_e/release_v2.html). Strain rates are estimated at 0.5 X 0.5 grid points; each curve represents the trade-off at one grid point. All the curves are concave, suggesting that one may find a spot along the curve “closest” to the origin. To do so we first normalize W and s by dividing them with their asymptotic offsets with the two axes respectively. We then measure the “distance” from the origin to the curve and choose the one with the shortest distance as our preferred solution. The high strain rates concentrate along the San Andreas, San Jacinto, and Imperial Valley faults, and at the 1992 Landers earthquake epicentral region. This is similar to what we have found before (e.g. Jackson et al. 1997) using the old interpolation method. However, we can identify finer details in some regions with dense station distribution than in previous results, demonstrating the capacity of the method to better exploit the resolving power of the data.

References

- Jackson, D. D., Z.-K. Shen, D. Potter, X. Ge, L. Sung, Southern California deformation, *Science*, **277**,1621-1622, 1997.
- Shen Z.-K., D. D. Jackson, and B. X. Ge, Crustal deformation across and beyond the Los Angeles basin from geodetic measurements, *J. Geophys. Res.*, **101**, 27,957-27,980, 1996.

Publication resulted from this project

- Shen, Z.-K., and D. D. Jackson, Optimal estimation of geodetic strain rates from GPS data, *EOS Trans. AGU*, **81**, No.19, p.S406, 2000.