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New Methodology in Computational Seismology for Dynamic Rupture along Complex Fault Systems

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The main concern of this project has been to develop new spectral computational methods for fully inertial, continuum elastodynamic, rupture analyses. The methods are needed for studies of earthquake nucleation and propagation. In the past year, our special focus has been on ruptures along dissimilar material interfaces. The problem has been studied numerically, with comparison of formulations, and the ill-posed nature of certain versions of this problem have been addressed, along with possible regularizations.

2D and 3D spectral formulations for dissimilar materials: The coupling between slip and normal stress that exists along an interface separating materials with different elastic properties allows for unstable propagation of rupture even with initial stress conditions that fall below the Coulomb threshold, a possibility noted by Weertman [*JGR*, 1980]. Andrews and Ben-Zion [*JGR*, 1997] attempted to simulate numerically such problem with a finite difference method with 2D in-plane geometry. They indeed observe the propagation of a self-sustained pulse that propagates at the generalized Rayleigh wave speed (which exists for not too dissimilar materials). It is not a steady state pulse and soon splits into several pulses that have increasing amplitude. They also mention corruption of their results by grid discreteness. We attempted to reproduce Andrews and Ben-Zion's study to check the resolution problems, and to what extent the aforementioned pulse properties were due to numerical noise, by using a spectral boundary integral equation (BIE).

(i) ***Combined versus independent formulation (2D):*** The spectral method introduced by Perrin et al. [*J. Mech. Phys. Solids*, 1995] and Geubelle and Rice [*ibid*, 1995] for homogeneous materials had been so far implemented in the following way: at each point of the interface, the wave mediated stress contribution from the whole fault is evaluated, computing a convolution integral, by taking into account the displacements *discontinuities* at the interface. The convolution integrals thus involve kernels that takes into account the *combined* effect of the two half-spaces. Hence, this implementation has been named the ‘combined formulation’.

When applied to dissimilar materials, these combined kernels are more complicated to obtain and to deal with. Geubelle and Breitenfeld [*Int. J. Fracture*, 1997] introduced a so-called ‘independent formulation’ for which the *actual* displacements at the interface are used, the convolution kernels dealing with each half-space *independently*. The kernels' expressions are much simpler but in counterpart there are twice as many operations to perform. Geubelle and Breitenfeld developed both formulations in the 2D mode III case and concluded that these were equivalent in terms of stability and accuracy.

For the in-plane case, Breitenfeld and Geubelle [*ibid*, in press] developed the independent formulation only. They compared it with the combined formulation, for homogeneous materials only, for various versions of the Lamb problem. Contrary to the anti-plane case, they observe that the independent formulation is more stable, giving less numerical oscillations.

The homogeneous case being a particular case of the dissimilar case, the above property is expected to remain true in the dissimilar case. However, we decided to develop both the combined and independent formulations for the in-plane case. In addition of providing an independent confirmation of Geubelle et al.'s analysis, we wanted to check it for cases more realistic than the Lamb problem. Further it gives a good way of checking computer codes, which proved very useful for dissimilar material for which no analytical solution exist.

For the combined formulation, the expressions for the convolution kernels in the Laplace domain are *huge* (several pages of algebra for each of the 3 kernels). However actual kernels are obtained by numerical inversion of Laplace transforms, which need to be done only once, the results being stored in computer files.

We compared the independent and combined formulations for the case of non-propagating cracks submitted to an instantaneously and uniformly applied stress load. We reached the same conclusion as Geubelle's group's: the independent formulation is more stable than the combined one. [Why this is so, contrary to the anti-plane case, is still an open question.]

Consequently, we have used mainly the independent formulation but is it the best choice, always? This is not necessarily true since (a) the bad stability might be cured for cases for which a physical stabilization exist (e.g., when realistic constitutive laws, with regularizing length scale(s) are used); (b) by design, with the independent formulation, and contrary to the combined one, avoiding spatial replications inherent to the basic spectral method [Cochard and Rice, *J. Mech. Phys. Solids*, 1997] is not computationally advantageous. For these reasons, the combined formulation might still be useful in, e.g., the study of earthquake sequences, including long, quasi-static periods (versus study of a single, fully dynamic event). Finally the combined formulation can be very useful in some cases, not explained here, for which the independent formulation cannot be easily used.

(ii) ***Numerical stability and the delay parameter:*** the convolution integrals are in the form $f(t) = \int_0^t K(t-t')D(t')dt'$, where K is a convolution kernel and D is the displacement or displacement discontinuity in the Fourier domain. It has been shown in the past for the anti-plane case [Morrissey and Geubelle, *Int. J. Numer. Methods Eng.*, 1997; Cochard and Madariaga, *Pure Appl. Geophys.*, 1994] that introducing an artificial delay, i.e., using $f(t) = \int_0^t K(t-t'+delay)D(t')dt'$ instead, where *delay* is a fraction of the discretized time step (used when computing the integral numerically), led to much more stable numerical results.

We first reached an identical conclusion for the in-plane mode II case. The test studies had been done with homogeneous materials and simple cases, for which, although no analytical results could be compared with, reasonable confidence could be reached, by having convergence through grid size reduction and by comparing with results obtained with the cellular implementation of the BIE method. At first sight, the conclusion appeared to remain valid for dissimilar materials as well, use of a delay leading to more stable numerical results. However, we finally realized that the cases tested with dissimilar materials where indeed physically unstable (see below). Use of a delay in fact provides an *artificial* stabilization. A wiser conclusion is then that such delay must be used with great care! Results presented below have been obtained without delay.

(iii) ***Results:*** Taking all parameters identical to those used by Andrews and Ben-Zion [1997], we find similar grid resolution problems; the number and amplitude of the pulses observed at a given time changed through each reduction of grid size by a factor of 2 that we tried, with no hint of convergence (see fig. 1). In fact, Adams [*J. Appl. Mech.*, 1995] precisely exhibited the unstable nature of the frictional bi-material interface, showing that the problem involving 2 elastic half-spaces of different material properties slipping at a constant velocity with constant friction coefficient *f* is, for a wide range of parameters, unstable under a perturbation of any wavelength. The smaller the wavelength, the faster the occurrence of the instability (with exponential growth in time). The spatial frequency spectrum of any real problem will contain all wavelengths, including arbitrarily small ones. As a consequence, the perturbation of stress or velocity at a given point of the interface will become arbitrarily large at arbitrarily small time. We thus realized that the problem is ill posed, unless some regularization is performed (discussed below).

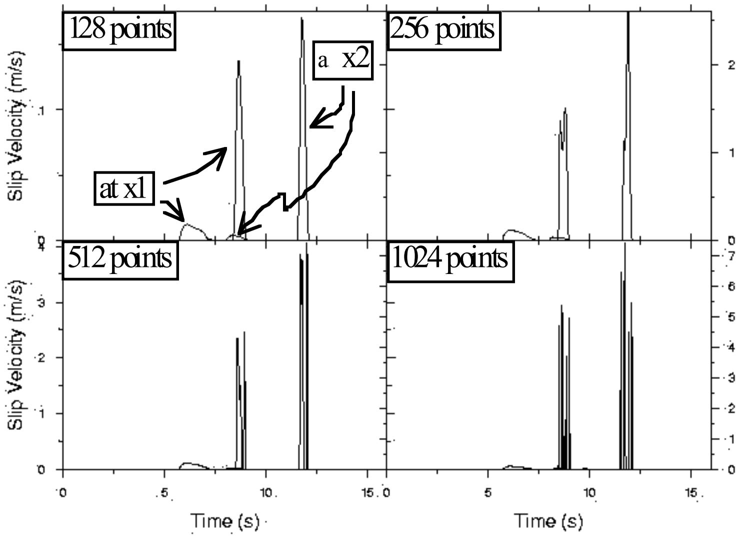

Figure 1: Slip velocity as a function of time at 2 positions x1 and x2 along an interface between dissimilar materials, for 4 different grid resolutions. Rupture has been artificialy initiated elsewhere on the fault plane. All physical parameters are identical to those used by Andrews and Ben-Zion [1997] in their fig. 11 (x1 and x2 correspond to their 6 and 8.1, respectively). The grid resolution problems illustrate the *physical* nature of the instability.

Applied to a single frequency stress perturbation, our numerical method nicely reproduces Adams' stability predictions, the amplitude of the perturbation growing in the unstable cases while decreasing in the stable ones. This is thus an encouraging test for our numerical formulation. We can still simulate physical problems that lay out of the unstable range and in such cases we indeed obtain convergence through grid size reduction. For material mismatches large enough so that the generalized Rayleigh wave speed no longer exists, Adams showed that there exist a transition in terms of the value of the *f*, larger *f* still leading to unstable behavior, but smaller ones giving rise to stable behavior. We simulated various material contrasts (ranging from 30% to 100%). Although we choose an initial shear stress just below the frictional strength, we do not generate self-sustained pulses, rather decaying ones [Cochard, *EOS*, 1998]. Also, the velocity at which those pulses propagate is not clearly defined. E.g., in the 30% material contrast case, we observe two such pulses, propagating at the smaller shear and dilational wave speeds, but for other mismatches, there is just one pulse whose velocity corresponds to none of the identified wave speeds.

To stay in the range of contrasts that allow for a generalized Rayleigh wave speed to exist, and hence for which the physical problems are Adams-unstable for all values of *f*, or to deal with the unstably high range of *f* in other cases, we need to regularize the problem. A possible route is to modify the constitutive law by introducing some length scale. In view of the coupling between slip and normal stress, it is necessary to use friction laws that allow for variable normal stress. These have been provided based on experiments with jumps in normal stress by Linker and Dieterich [*GRL*, 1992] and Dieterich and Linker [*GRL*, 1992], although there is not full consistency between their description of the process and the results found by Prakash and Clifton [*Soc. Exper. Mech.*, 1992] and Prakash [*J. Tribology*, 1998] for sudden large normal stress changes due to a reflected shock waves during oblique impact. We first studied a simplified version of the Prakash-Clifton law, namely $d\tau / dt = -(V / L)(\tau - f\sigma)$, where V is the sliding velocity and L is the state evolution length. In the spirit of Adams [1995], we studied the consequences of a suddenly imposed shear stress perturbation, in the form $Q(t)$ exp(ikx), where $Q(t)$ is arbitrary, on a sliding interface. We have been able to show in theoretical work (done in collaboration with K. Ranjith), as well as in our numerical simulations, that such constitutive description indeed provides the expected short-wavelength regularization (see fig. 2).

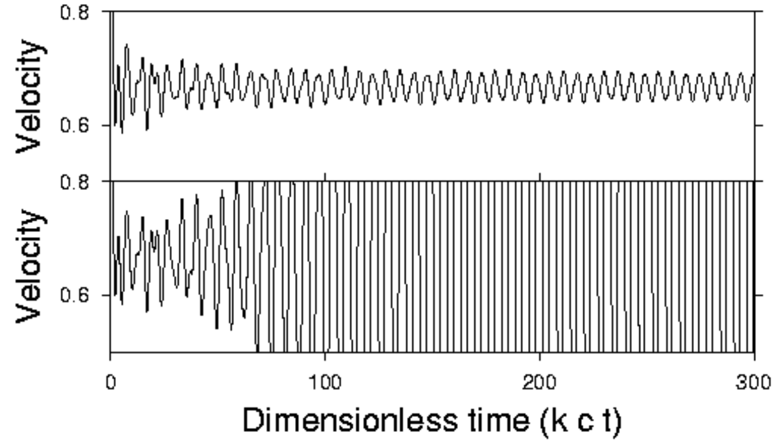

Figure 2 : Normalized slip-velocity versus normalized time at a point of a bi-material interface submitted, at time 0, to an instantaneous, spatially periodic stress perturbation of wavenumber k. Bottom: without regularization. Top: with the Prakash-Clifton-like friction law as a regularization: the divergence has been suppressed.

(iv) ***Extension to 3D vectorial elastodynamics:*** as usual with the spectral method, the generalization to 3D is rather straightforward, the convolution kernels being the same as for the 2D case. We have done it and have checked our codes for various cases in the 2D limit. Recently we have also tried to compare the seismograms generated at a bi-material interface by a moment point source, as analytically derived by Ben-Zion [*GJI*, 1989]. We are able to reproduce the early parts of the seismograms, which is very encouraging regarding the difficulties met when a finite element method is used [Ben-Zion, *personal communication*, 1998]. However, the displacement finally becomes infinitely large. It turned out that this numerical stability problem is due to the nature of the loading: the *displacement* is imposed at the source in this case whereas, most of the time, the fault is loaded with an imposed *stress*. We have not found a cure for this problem yet and are still investigating it.

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